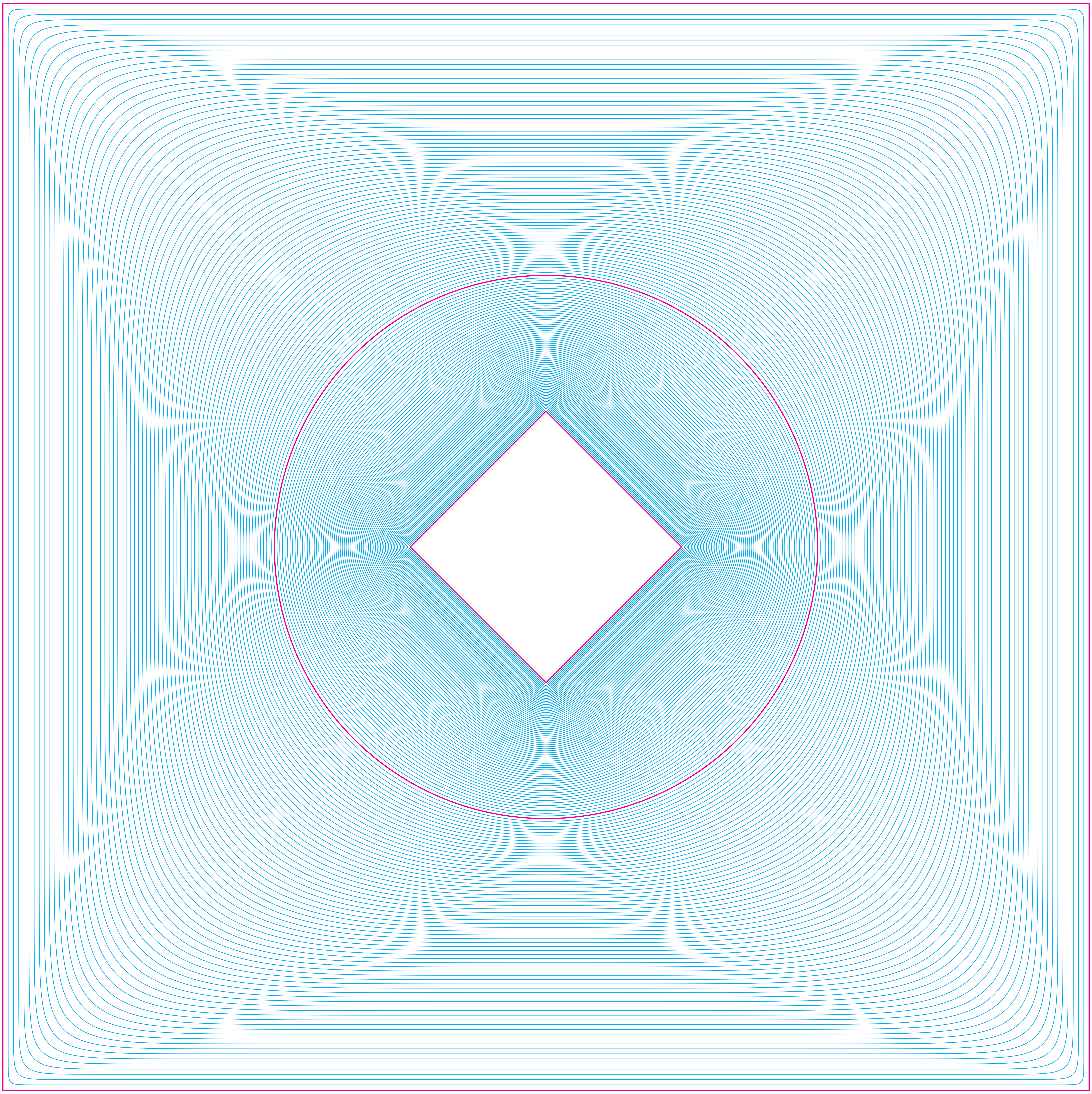
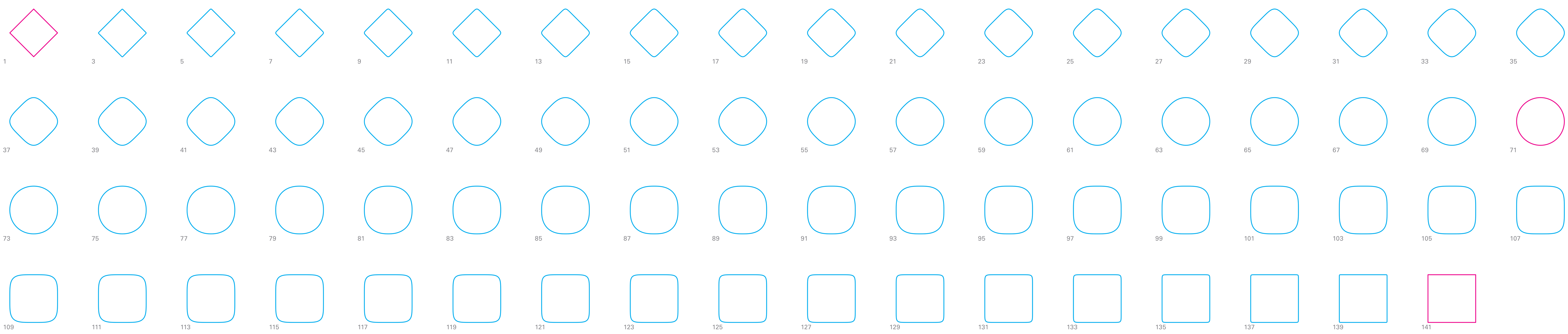




The space of shapes between square, circle, and square

This project grew out of questions about how to draw nested ovals so that they appear consistent in the steps from one to the next. That led to discussions about ovals, which led to discussions about super ellipses, which led to discussions about the space of shapes between circle and square. We turned to our friend Patch Kessler who figured out the math and visualized these for us.

There are 70 closed curves outside the circle, the outermost of which is a square. There are another 70 closed curves inside the circle, the innermost of which is a square. Including the circle, there are 141 total shapes. The main shapes—circle, and inner and outer squares—are thicker and magenta. The closed curves in between the main shapes are thinner and cyan.

Underneath, every other shape has been placed in rows so you can see the shapes transitioning from left to right, like a storyboard.

Gavin Miller created an interactive tool for generating variations and exporting SVGs. https://curves.dubberly.com/

When $p = 2$, the following Lamé curve is a circle with diameter $2r$.

$$\left(\frac{x}{r}\right)^p + \left(\frac{y}{r}\right)^p = 1$$

As $p \to \infty$ this curve approaches a square with side length $2r$. We have been exploring collections of these curves with p and r chosen so that a gradual transition occurs from square to circle to square. We maintain $p \geq 2$, both inside and outside the circle, because although there is a transition from circle to square as p goes from 2 to 1, we find that it doesn't have nice corners.

We label the circle with index $k = 0$, and we count up curve by curve to $k = n$ for the outer square and down curve by curve to $k = -m$ for the inner square. We build curve k so that it intersects the y-axis a distance $e^{\alpha k}$ from the origin, and so that it intersects the 45° line a distance $e^{\beta k}$ from the origin.

In order for curves $-m$ and n to be squares, we need $m = n$ and

$$\beta - \alpha = \frac{\ln \sqrt{2}}{n}$$

If we've chosen n, and we want

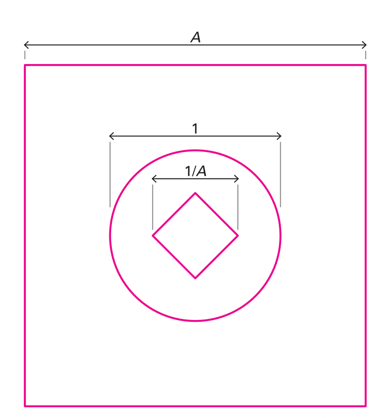
$$\frac{\text{outer square side length}}{\text{circle diameter}} = A$$

for some $A > 1$, it follows that

$$\alpha = \frac{\ln A}{n} \qquad \beta = \frac{\ln A}{n} + \frac{\ln \sqrt{2}}{n}$$

and

$$\frac{\text{inner square diagonal}}{\text{circle diameter}} = \frac{1}{A}$$



In the artwork on this poster A is equal to 2.

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